

Wilson Loops in Open String Theory

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Abstract

Wilson loop elements on torus are introduced into the partition function of open strings as Polyakov's path integral at one-loop level. Mass spectra from compactification and expected symmetry breaking are illustrated by choosing the correct weight for the contributions from annulus and Möbius strip. We show that Jacobi's imaginary transformation connects the mass spectra with the Wilson loops. The application to thermopartition function and cosmological implications are briefly discussed.

The string theory [1] as a candidate for a unified model of interactions contains gauge fields with a large symmetry. The gauge symmetry must be broken down to more or less “realistic” symmetry, such as $SU(3) \times SU(2) \times U(1) \times \dots$, in an early stage of the universe. A symmetry breaking mechanism has been proposed, which is called the Wilson loop mechanism.[2] Roughly speaking, it can be said that the gauge fields with non-zero vacuum expectation values (up to gauge transformations) on a non-simply connected space play the role of Higgs fields in a field theoretical perspective.

Some simple models were investigated in the framework of Kaluza-Klein theory.[3] An important point is that the energies of vacua corresponding to different symmetries are degenerate at classical level.[3, 4] The vacuum energy from one-loop quantum effect must be taken into consideration in order to determine the “true” vacuum. The calculation was made at zero temperature as well as finite temperature.[5, 6] In order to evaluate the free energies in Kaluza-Klein theory, we need to know the excitation modes which arise from the compactification of the extra space. In the presence of the vacuum gauge fields, such mass spectra might shift up or down according to the magnitude of the vacuum gauge fields and the coupling to the field. The vacuum and free energies are obtained by using the mass spectra and they are expressed as a function of the vacuum gauge fields.

On the other hand, in string theories, the vacuum energy or cosmological constant is computed in Polyakov's path integral method most clearly.[7] The free energy at finite temperature can also be obtained in path integral form, as

is shown by Polchinski in the case of bosonic strings.[7] The path integral approach is also useful for investigation of torus compactifications with background fields.[8]

In this paper, we will show how the vacuum gauge fields on torus modify the mass spectra in open string theory in terms of the path integral method. Polyakov's path integral formalism for the open string is developed by several authors.[9] For our purpose, we have only to consider the zero-mode pieces of bosonic string coordinates and the background gauge fields. In other words, the "Kaluza-Klein" excitation is added to each "stringy" mass level. Since the generalization to the superstring is straightforward, we start with the following world-sheet action for the bosonic strings, which was given by Callan et al.[10] as

$$S = \frac{T}{2} \int d^2\sigma \sqrt{g} g^{ab} \partial_a X^M \partial_b X_M + iT \oint ds A_M \frac{\partial}{\partial s} X^M, \quad (1)$$

where X^M are bosonic fields and T is the string tension. The line integral is assumed to be path-ordered on the boundary of the world sheet. We can take the world-sheet metric g_{ab} for annulus, such that

$$d^2\sigma = g_{ab} d\sigma^a d\sigma^b = d\sigma_1^2 + t^2 d\sigma_2^2 \quad (0 \leq \sigma_1 \leq 1, 0 \leq \sigma_2 \leq 1), \quad (2)$$

where t is the moduli parameter. (For Möbius strip and other configurations, parametrization of g_{ab} is given in Ref. [9].)

From now on, we only consider the world sheet configurations which have boundaries, since the gauge field is attached to boundaries. The traces of two ends of an open string correspond to boundaries of the world sheet at $\sigma_1 = 0$ and $\sigma_1 = 1$.

For simplicity, we consider $SO(N)$ as the gauge group and a torus in one space dimension. Then A_M is $N \times N$ matrix valued.

In this compact dimension, denoted by I -th dimension, string coordinate field can be written as

$$X^I = x^I + 2\pi r \ell \sigma_2 + (\text{oscillators}), \quad (3)$$

where ℓ is an integer. The oscillator part is expressed in a function periodic in σ_2 .

We set the radius of the torus to r in Eq. (3). This means that the points separated by $2\pi r$ in the I -th direction are identified to a point on the torus.

We find that substitution of the zero-mode piece in Eq. (3) into the action (1) yields the Wilson loop element; the integer ℓ indicates the winding number of the Wilson line around the torus.

The partition function in the path integral form is proportional to the factor

$$f = \sum_{\ell=-\infty}^{\infty} \exp \left[-\frac{(2\pi r)^2 T \ell^2}{2t} \right] \prod_{\text{boundaries}} \text{Tr} \exp(i2\pi r \ell A_I L_B), \quad (4)$$

where L_B is the length of the boundary. An annulus has two boundaries with $L_B = 1$ while a Möbius strip has one boundary with $L_B = 2$.

Now we study the mass spectra which come from the compactification and the nature of the symmetry breaking induced by A_I . For concreteness, we take A_I as the following $N \times N$ matrix

$$A_I = A \begin{bmatrix} 0 & -i & 0 & \cdots \\ i & 0 & 0 & \cdots \\ 0 & 0 & 0 & \\ \vdots & \vdots & & \ddots \end{bmatrix}. \quad (5)$$

Then we get the factors for an annulus and a Möbius strip respectively as:

For an annulus,

$$f_A = \sum_{\ell=-\infty}^{\infty} \exp \left[-\frac{(2\pi r)^2 T \ell^2}{2t} \right] \cdot [N + 2\{\cos(2\pi\phi\ell) - 1\}]^2; \quad (6)$$

For a Möbius strip,

$$f_M = \sum_{\ell=-\infty}^{\infty} \exp \left[-\frac{(2\pi r)^2 T \ell^2}{2t} \right] \cdot [N + 2\{\cos(4\pi\phi\ell) - 1\}], \quad (7)$$

where $\phi \equiv rTA$.

The weight for each contribution must be specified when we sum up these two. We can determine it by comparing the trivial case, $\phi = 0$, with the known result.[9] For the open bosonic and superstring, massless modes of “stringy” excitation must contribute to the vacuum amplitude with degeneracy $N(N - 1)/2$, the degree of freedom in the adjoint representation. The Kaluza-Klein mode is added to each stringy excitation, which is labelled by an occupation number of oscillators.

Let us consider the excitation for each stringy mode. It is well known that the relative sign of contributions from annulus and Möbius strip is alternating according to even and odd mode.

To get the Kaluza-Klein mass spectra, we use Jacobi’s imaginary transformation such as, in our case,

$$\sum_{\ell=-\infty}^{\infty} \exp \left[-\frac{(2\pi r)^2 T \ell^2}{2t} \right] \cdot \cos(2\pi\phi\ell) = \sqrt{\frac{t}{2\pi T r^2}} \sum_{\ell=-\infty}^{\infty} \exp \left[-\frac{\pi t(\ell - \phi)^2}{2\pi T r^2} \right]. \quad (8)$$

Then we find the following results. For even modes, which include the massless mode, the integrand contains the factor

$$\begin{aligned} \frac{f_A - f_M}{2} &= \sqrt{\frac{t}{2\pi T r^2}} \cdot \left[[(N - 2)(N - 3)/2 + 1] \cdot \sum_{\ell} \exp \left[-\frac{\pi t \ell^2}{2\pi T r^2} \right] \right. \\ &\quad \left. + 2(N - 2) \cdot \sum_{\ell} \exp \left[-\frac{\pi t(\ell - \phi)^2}{2\pi T r^2} \right] \right], \end{aligned} \quad (9)$$

while for odd modes, it contains the factor

$$\begin{aligned}
\frac{f_A + f_M}{2} = & \sqrt{\frac{t}{2\pi T r^2}} \cdot \left[[(N-2)(N-1)/2 + 1] \cdot \sum_{\ell} \exp \left[-\frac{\pi t \ell^2}{2\pi T r^2} \right] \right. \\
& + 2(N-2) \cdot \sum_{\ell} \exp \left[-\frac{\pi t (\ell - \phi)^2}{2\pi T r^2} \right] \\
& \left. + 2 \cdot \sum_{\ell} \exp \left[-\frac{\pi t (\ell - 2\phi)^2}{2\pi T r^2} \right] \right], \tag{10}
\end{aligned}$$

It is known that t will be the Schwinger parameter up to rescaling by T if path integral is regarded as the integration of the heat-kernel method.[7, 9] The results agree with the fact that the even modes correspond to the fields in the adjoint representation of $SO(N)$ and the odd modes correspond to the fields in the symmetric representation of $SO(N)$; for each even mode, the shifts in the additional masses, expressed in the unit of ϕ here, are proportional to commutator of the vacuum gauge field and the corresponding field, while for each odd mode, those are proportional to anticommutator. The massless mode, which belongs to even modes, corresponds to the excitation of gauge fields. Then the gauge symmetry is broken to as $SO(N) \rightarrow SO(N-2) \times U(1)$ in the case considered here.

To summarize: we find that the correct mass spectra are obtained, through compactification, by means of the path integral expression with the world-sheet action for open strings which contains the non-zero background gauge field.

The result should be applied to computation of vacuum and free energy of open strings at one-loop level. In particular, it provides a useful relation in the investigation of the temperature-dependence of the energy. In field theory, the free energy obtained in Ref. [5] is expressed just in the expansion in terms of the trace of the Wilson loops. Finite temperature effect only modifies the “weight” for each Wilson loop. But the dominance of the Wilson loop of the lowest winding number does not undergo a change in a simple model. Thus we suspect that finite temperature effect does not have much influence on the Wilson-loop breaking in general.

As for the case of open strings, Wilson loops are directly incorporated in the calculation of the path integral. We can treat the symmetry breaking by Wilson loops in a lucid style. It is interesting to consider Wilson loop mechanism in the superstring. In supersymmetric theories, quantum effects are often cancelled between bosons and fermions, then the expansion with respect to Wilson loops might bring about trivial results. The supersymmetry is broken at finite temperature. The connection between the evolution of the hot universe and the symmetry breaking may be a matter of interest.

The study of Wilson loops in the heterotic strings [12] and the fermionic string theories [13] is required urgently. We hope to report on these subjects elsewhere.

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